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Modifications in the Stress Field of a Long Inclined Fault Caused by the Welded-Contact Across the Interface between Elastic Half-Space and Orthotropic Half-Space

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Abstract—*Some stress components are not required to be continuous across the welded-contact boundary between two elastic half-spaces. The welded-contact boundary conditions between a hard half-space and a soft half-space are responsible for major changes in the stress field. The stress component, which is not involved in the boundary conditions, is significantly higher along the hard side of the interface. Our aim of study is the modifications in the stress field of a long inclined dip-slip fault caused by the welded-contact boundary conditions across the interface between elastic half-space and orthotropic half-space. In the case of a dip-slip fault, the Poisson's ratio of the half-space in which the fault lies, has a significant influence on the stress field across the interface.*

Keywords*: Dip-Slip Fault, Stress Field, Welded-Contact.*

1. INTRODUCTION

The theory of elastic dislocation has been introduced by Steketee and Maruyama for the mathematical and physical description of mechanics of earthquakes. Savage and Rybicki gave the extensive reviews of the application of the elastic dislocation theory to earthquake faulting problems. Both two- and three-dimensional fault models have been used in the literature. Considering the fact that some of the faults are sufficiently long, the two-dimensional fault model is used.

Non-homogeinity of the Earth compels us to consider the effect of internal boundaries on the stress field generated by earthquake faults. The knowledge of the modification of the stress field caused by internal boundaries is useful to study secondary faulting. Bonafede and Rivalta obtained a plane strain analytic solution for the displacement and stress fields produced by a long vertical tensile dislocation in the proximity of the interface between two elastic half-spaces in welded contact. In a subsequent paper, Bonafede and Rivalta derived the corresponding solution for a long vertical tensile crack. They noted that the discontinuities in the elastic parameters across the boundary act as stress concentrators for the stress component not involved in the boundary conditions. Rivalta *et al.* provided a plane strain analytic solution for the displacement and stress fields induced by an edge dislocation in an elastic half-space in welded con-tact with another elastic half-space. Rybicki and Yamashita derived formulas for two-dimensional anti-plane and in-plane problems relating stresses across a plane boundary between two elastic half-spaces in welded contact, assuming a homogeneous shear stress in one of the two half- spaces. They concluded that the mechanical conditions related to faulting within the Earth's crust are expected to be favourable in the high rigidity media.

The welded-contact boundary conditions between a hard half-space and a soft half-space are responsible for major changes in the stress field. The stress component, which is not involved in the boundary conditions, is significantly higher along the hard side of the interface. We found the unexpected differences in the normal component of stress parallel to the interface which is not required to be continuous at the interface by the boundary conditions. This stress component shows wide regions of high stress in the harder side of the interface. Stress concentration along the interface is particularly high when the rigidity contrast is high.

Our aim of study is the modifications in the stress field of a long inclined dip-slip fault caused by the welded-contact boundary conditions across the interface between elastic half-space and orthotropic half-space. In the case of a dip-slip fault, the Poisson's ratio of the half-space in which the fault lies, has a significant influence on the stress field across the interface.

2. THEORY

Let the Cartesian co-ordinates be denoted by (x_1, x_2, x_3) with x_3-axis vertically upwards. Consider two homogeneous, perfectly elastic half-spaces which are welded along the plane $x_3 = 0$. The upper half-space $(x_3 > 0)$ is assumed to be isotropic with stress-strain relation

$$p_{ij} = 2\mu\left[e_{ij} + \frac{v_1}{1-2v_1}\delta_{ij}\, e_{kk}\right], (i,j = 1,2,3) \qquad (1)$$

where, p_{ij} are the components of stress tensor, e_{ij} are the components of strain tensor, μ is the shear modulus(rigidity) and v_1 is Poisson's ratio. The lower half-space ($x_3 < 0$) is assumed to be orthotropic with stress-strain relation.

$$\begin{bmatrix} p'_{11} \\ p'_{22} \\ p'_{33} \\ p'_{23} \\ p'_{31} \\ p'_{12} \end{bmatrix} = \begin{bmatrix} c_{11} & c_{12} & c_{13} & 0 & 0 & 0 \\ c_{12} & c_{22} & c_{23} & 0 & 0 & 0 \\ c_{13} & c_{23} & c_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & c_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & c_{55} & 0 \\ 0 & 0 & 0 & 0 & 0 & c_{66} \end{bmatrix} \begin{bmatrix} e'_{11} \\ e'_{22} \\ e'_{33} \\ 2e'_{23} \\ 2e'_{31} \\ 2e'_{12} \end{bmatrix} \qquad (2)$$

The plane strain problem for an isotropic medium can be solved in terms of Airy stress function U such that

$$p_{22} = \partial^2 U/\partial x_3^2,\ p_{33} = \partial^2 U/\partial x_2^2, p_{23} = -\partial^2 U/\partial x_2 \partial x_3 \quad (3)$$

$$\nabla^2\nabla^2 U = 0 \qquad (4)$$

The plane strain problem for an orthotropic medium can be solved in terms of the Airy stress function U^* such that Garg et al (1991)

$$p'_{22} = \partial^2 U^*/\partial x_3^2,\ p'_{33} = \partial^2 U^*/\partial x_2^2,\ p'_{23} = -\partial^2 U^*/\partial x_2 \partial x_3, \qquad (5)$$

$$\left(a^2\frac{\partial^2}{\partial x_2^2} + \frac{\partial^2}{\partial x_3^2}\right)\left(b^2\frac{\partial^2}{\partial x_2^2} + \frac{\partial^2}{\partial x_3^2}\right)U^* = 0 \qquad (6)$$

$$a^2 + b^2 = \frac{(c_{22}c_{33} - c_{23}^2 - 2c_{23}c_{44})}{c_{33}c_{44}}, a^2 b^2 = \frac{c_{22}}{c_{33}} \qquad (7)$$

Bala and Rani (2009) derived closed-form analytic expressions for the displacements and stresses caused by along inclined dip-slip fault located in an elastic half-space in welded contact with another orthotropic elastic half-space. The stress components p_{23} and p_{33} are continuous across the interface as required by the welded-contact boundary conditions. At the interface, the stress component p_{22} is not continuous. In Figure 1, the origin is taken at the fault trace, that is at the point where the fault, if extended, meets the interface $x_3 = 0$. At the origin, the stress component p_{22} is given by

For Isotropic half − space,

$$p_{22} = \frac{\alpha\mu b}{\pi}\left[\sin 2\delta\left\{\frac{1 + X_1 - 4X_2}{s}\right\}\right]\Bigg|_{L_1}^{L_2} \qquad (8)$$

For Orthotropic half-space

$$p'_{22} = \frac{2\alpha\mu b}{\pi}\left[\sin 2\delta\left\{\frac{a^2 A + b^2 B}{s}\right\}\right]\Bigg|_{L_1}^{L_2} \qquad (9)$$

where, $\alpha = 1/2(1 - v_1)$, $X_1 = 2(A + B) - 1$, $X_2 = A(1 + a) + B(1 + b) - 1$,

$$A = \frac{\alpha(1 - b + 2\mu s_2 - 2\mu r_2)}{W}, B = \frac{\alpha(a - 1 + 2\mu r_1 - 2\mu s_1)}{W}$$

$$r_1 = \frac{c_{33}a^2 + c_{23}}{\Delta}, r_2 = \frac{c_{33}b^2 + c_{23}}{\Delta},$$

$$s_1 = \frac{c_{23}a^2 + c_{22}}{a\Delta}, s_2 = \frac{c_{23}b^2 + c_{22}}{b\Delta}, \Delta = (c_{22}c_{33} - c_{23}^2)$$

$$W = (1 + a - \alpha + 2\mu\alpha r_1)(1 + b - b\alpha + 2\mu\alpha s_2) - (1 + a - a\alpha + 2\mu\alpha s_1)(1 + b - \alpha + 2\mu\alpha r_2),$$

δ is the dip angle, b is the fault-slip and L_1, L_2 are shown in Figure 1. Therefore, at any point of the interface,

$$K = \frac{p'_{22}}{p_{22}} = \frac{2(a^2 A + b^2 B)}{1 + X_1 - 4X_2} \qquad (10)$$

It may be noted that the stress ratio is independent of dip angle δ and depends on the elastic constants c_{22}, c_{23}, c_{33}, c_{44} of the orthotropic half-space and μ (rigidity) and v_1 (poisson's ratio) of isotropic half-space.

Here we want to see the stress ratio variation with rigidity ratio c_{44}/μ for different values of poisson's ratio, where c_{44} is the rigidity of orthotropic half-space. Also we want to compare the stress ratio between the model having orthotropic half-space as Topaz and Barytes.

Figure 2 represents the variation of stress ratio with rigidity ratio for v_1= 0.1, 0.2, 0.3, 0.4 (a) $0 \leq c_{44}/\mu \leq 1$ (b) $1 \leq c_{44}/\mu \leq 2$, for the case if we consider the orthotropic half-space as Topaz.

In figure 2(a), it is observed that in the range $0 \leq c_{44}/\mu \leq 1$, K <1. Also for a given c_{44}/μ, K decreases as v_1 decreases. For c_{44}/μ near to 1, stress ratio increases fast for v_1= 0.4.

In figure 2(b), for $1 \leq c_{44}/\mu \leq 2$, K<1 for v_1= 0.1, 0.2. But for

$$\frac{c_{44}}{\mu} > 1.5, K > 1\ v_1 = 0.3$$

and for $\frac{c_{44}}{\mu} > 1.1, K > 1\ v_1 = 0.4$.

Figure 3 represents the variation of stress ratio with rigidity ratio for v_1= 0.1, 0.2, 0.3, 0.4 (a) $0 \leq c_{44}/\mu \leq 1$ (b) $1 \leq c_{44}/\mu \leq 1.8$, for the case if we consider the orthotropic half-space as Barytes.

In figure 3(a), for $0 \leq c_{44}/\mu \leq 1$,

$0 \leq K \leq 1.5$ for v_1= 0.1, 0.2; $0 \leq K \leq 2$ for v_1= 0.3; $0 \leq K \leq 3$ for v_1= 0.4.

In figure 3(b), for $0 \leq c_{44}/\mu \leq 1$,

$0 \leq K \leq 4$ for v_1= 0.1, 0.2, 0.3;

but for v_1= 0.4, the value of K shows the variation more significantly.

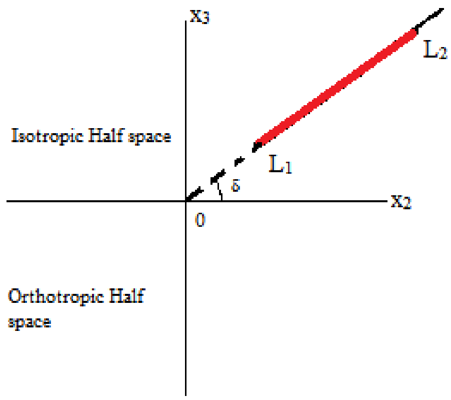
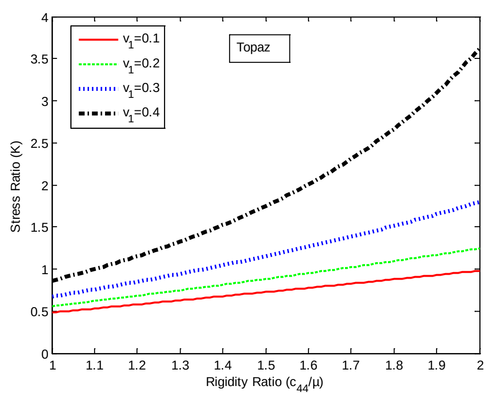
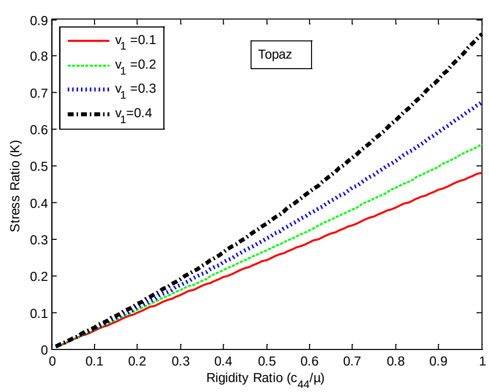


Figure 1: Geometry of a long dip slip fault lying in an elastic half-space in welded contact with orthotropic half-space. The x_1 -axis is taken parallel to the length of the fault and x_3 -axis normal to the interface between the two half-spaces. δ is the dip angle and L_1 , L_2 are the distances of the two edges of the fault from the origin.



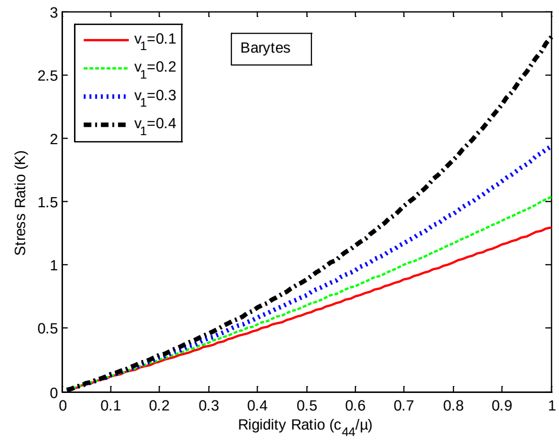
(a)



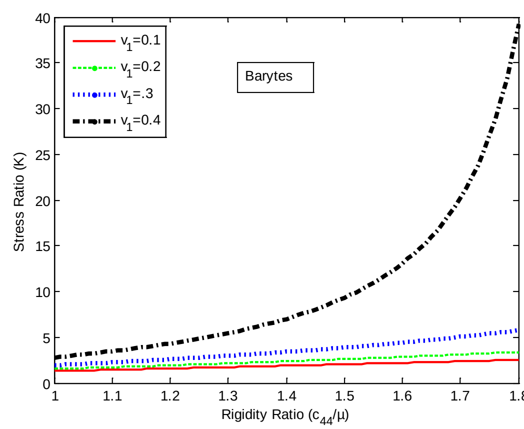
(b)

Figure 2. Variation of the stress ratio K with the rigidity ratio for four values of the Poisson's ratio of the isotropic half-space for (a) $0 \leq c_{44}/\mu \leq 1$ (b) $1 \leq c_{44}/\mu \leq 2$ for a long dip-slip fault lying in isotropic half space welded with Topaz.

Now if we compare between Topaz and Barytes with respect to stress ratio, the value of K more significantly increases in case of barytes in comparison to topaz, for given value of poisson's ratio and for rigidity ratio. This shows that the stress in case of barites is more significant than topaz.



(a)



(b)

Figure 3. Variation of the stress ratio K with the rigidity ratio for four values of the Poisson's ratio of the isotropic half-space for (a) $0 \leq c_{44}/\mu \leq 1$ (b) $1 \leq c_{44}/\mu \leq 2$ for a long dip-slip fault lying in isotropic half space welded with Barytes.

3. CONCLUSION

We have studied the modifications in the stress field of a two-dimensional inclined dip-slip caused by the welded-contact boundary conditions across the boundary between the two elastic half-spaces. It is assumed that the boundary between the half-spaces is taken as the x_3 = 0 plane and the fault is striking in the x_1 direction. The normal stress p_{33} and the shear stress p_{23} are required to be continuous. There is no restriction on the normal stress p_{22}. We find that the ratio of the normal stress p_{22} at the interface when approached from the two half-spaces depends on the rigidity ratio and the Poisson's ratio of the half-space in which the fault lies. It is independent of the dip angle of the fault. The most favourable elastic conditions for generation of large stresses near the interface in the other half-space occur when the dip-slip fault lies in the softer half-space with small compressibility.

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